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Elastic νe^- scattering of solar neutrinos with electromagnetic moments

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We consider the azimuthal asymmetry of the recoil electrons in elastic νe^- scattering of solar neutrinos, which can arise if neutrinos have electromagnetic moments and there is a large solar magnetic field. We show that using this effect it is not possible to distinguish between magnetic and electric dipole moments in the 1-Dirac and 2-Majorana neutrino cases and that averaging over neutrino energy is important and suppresses the azimuthal asymmetry in the 2-Majorana neutrino case.

[3] Neutrinos produced in the sun provide an interesting possibility to investigate neutrino properties. Beside the well-established neutrino oscillation search using solar neutrinos it is also well-known that they can be used to look for a magnetic moment (MM) or an electric dipole moment (EDM) of the neutrinos [1]. If there exists a large magnetic field inside the sun the so-called Resonant Spin-Flavour Precession scenario provides an appealing sollution to the solar neutrino problem [2]. Here we consider elastic scattering of solar neutrinos with MMs and EDMs off electrons. If the neutrinos acquire a transverse polarization because of the solar magnetic field there can be an azimuthal asymmetry in the recoil electron momentum [3]. Such an effect could be observed in a νe^- scattering experiment sensitiv to low energy solar neutrinos with good angular resolution like the proposed experiment HELLAZ [4]. Details of our considerations and further references can be found in Ref.[5].

The electromagnetic Hamiltonian. To describe the interaction of Dirac neutrinos with a MM and an EDM with the electromagnetic field we use the Hamiltonian

$$\mathcal{H}^D_{\rm em} = \frac{1}{2}\bar\nu_R \lambda \sigma^{\alpha\beta} \nu_L F_{\alpha\beta} + \text{h.c.} \tag{1}$$

Here $\nu^T_{L(R)} = (\nu_e, \nu_\mu, \nu_\tau, \nu_s, \ldots)_{L(R)}$ is the vector of the left-handed (right-handed) flavour eigenfields including an arbitrary number of sterile neutrinos. The hermitian matrices μ of MMs and d of EDMs are condensed in the non-hermitian matrix

$$\lambda = \mu - id \;\text{ with }\; \mu = \frac{\lambda+\lambda^\dagger}{2}\,,\; d = \frac{i(\lambda-\lambda^\dagger)}{2}\,. \tag{2}$$

If the basis of the neutrino fields is changed by unitary rotations $\nu_L = S_L \nu'_L$ and $\nu_R = S_R \nu'_R$ the matrix (2) in the new basis is obtained by the simple relation $\lambda' = S^\dagger_R \lambda S_L$, whereas μ and d obey rather complicated transformation laws [5].

Similarly, for Majorana neutrinos we have the Hamiltonian [6]

$$\mathcal{H}^M_{\rm em} = -\frac{1}{4}\nu^T_L C^{-1} \lambda \sigma^{\alpha\beta} \nu_L F_{\alpha\beta} + \text{h.c.}\,, \tag{3}$$

where C is the charge conjugation matrix. Now the matrix λ, defined as in Eq.(2), is antisymmetric and the MM and EDM matrices are antisymmetric and hermitian.

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The cross section. In addition to the electromagnetic interaction there is also the weak interaction of neutrinos with electrons, which is described by the Hamiltonian

$$\mathcal{H}_{\rm w} = \frac{G_F}{\sqrt{2}} \sum_\alpha \bar{\nu}_\alpha \gamma_\lambda (1-\gamma_5) \nu_\alpha \bar{e} \gamma^\lambda (g_V^\alpha - g_A^\alpha \gamma_5) e \,, \quad (4)$$

where G_F is the Fermi constant and $g_V^\alpha = 2\sin^2\Theta_W + d^\alpha$, $g_A^\alpha = d^\alpha$, $g_{V,A}^s = 0$, with the weak mixing angle Θ_W and $d^e = 1/2$, $d^{\mu,\tau} = -1/2$.

In the general case of an arbitrary neutrino polarization the cross section for elastic νe^- scattering consists of three terms

$$\frac{d^2\sigma}{dTd\phi} = \frac{d^2\sigma_{\rm w}}{dTd\phi} + \frac{d^2\sigma_{\rm em}}{dTd\phi} + \frac{d^2\sigma_{\rm int}}{dTd\phi} \,, \quad (5)$$

where ϕ is the azimuthal angle which is measured in the plane orthogonal to the momentum of the initial neutrino and T is the recoil energy of the scattered electron. The first and the second term are the pure weak and electromagnetic terms, respectively, and the third term is the interference term between the weak and the electromagnetic amplitude which is proportional to the transverse neutrino polarization and gives rise to the azimuthal asymmetry.

For the initial neutrino state in (5) we use an arbitrary superposition of flavour and helicity states:

$$|\nu\rangle_{\rm in} = \sum_{\alpha=e,\mu,\tau,s,\ldots} \left(a_-^\alpha |\nu_\alpha^{(-)}\rangle + a_+^\alpha |\nu_\alpha^{(+)}\rangle \right) \,. \quad (6)$$

In the massless limit the negative helicity states are left-handed neutrinos whereas the positive helicity states are sterile right-handed neutrinos in the Dirac case and right-handed antineutrinos in the Majorana case.

The weak cross section for Majorana neutrinos is given by

$$\frac{d^2\sigma_{\rm w}^M}{dTd\phi} = \sum_\alpha \left(|a_-^\alpha|^2 \frac{d^2\sigma_{\nu_\alpha e}}{dTd\phi} + |a_+^\alpha|^2 \frac{d^2\sigma_{\bar{\nu}_\alpha e}}{dTd\phi} \right) \,, \quad (7)$$

where $\sigma_{\nu_\alpha e\,(\bar{\nu}_\alpha e)}$ is the cross section for elastic scattering of neutrinos (antineutrinos) of flavour α off electrons given e.g. in [5]. For Dirac neutrinos the second term in Eq.(7) is absent.

The electromagnetic cross section has the same form for Dirac and Majorana neutrinos:

$$\frac{d^2\sigma_{\rm em}}{dTd\phi} = c \left(\frac{1}{T} - \frac{1}{\omega} \right) \left(a_-^\dagger \lambda^\dagger \lambda a_- + a_+^\dagger \lambda \lambda^\dagger a_+ \right) \,, \quad (8)$$

where $c = \alpha^2/2m_e^2\mu_B^2$, ω denotes the neutrino energy and $a_\mp^T = (a_\mp^e, a_\mp^\mu, \ldots)$.

The interference cross section is given by

$$\frac{d^2\sigma_{\rm int}}{dTd\phi} = f\, \mathrm{Re} \left[a_+^\dagger (\lambda g + \bar{g}\lambda) a_- (p_x' - i p_y') \right] \,. \quad (9)$$

Here we have defined $f = G_F \alpha / 2\sqrt{2}\pi m_e T \mu_B$ and $g = \mathrm{diag}\,[g_V^\alpha(2 - T/\omega) + g_A^\alpha T/\omega]$. For Majorana neutrinos $\bar{g} = \mathrm{diag}\,[g_V^\alpha(2 - T/\omega) - g_A^\alpha T/\omega]$ whereas $\bar{g} = 0$ in the Dirac case. For the direction of the initial neutrino we choose the z-axis and the transversal components of the recoil electron momentum are related to the azimuthal angle via $p_x' = p_\perp' \cos\phi$, $p_y' = p_\perp' \sin\phi$ with ${p_\perp'}^2 = {p_x'}^2 + {p_y'}^2$. Therefore we find from Eq.(9)

$$\frac{d^2\sigma_{\rm int}}{dTd\phi} \propto \cos(\phi - \gamma) \,. \quad (10)$$

The measurement of the azimuthal asymmetry in an experiment would allow to determine the angle γ which is defined as $\gamma \equiv \mathrm{Arg}[a_+^\dagger(\lambda g + \bar{g}\lambda) a_-]$.

In the following we will consider the question: *Is it possible to obtain information on complex phases in the electromagnetic moment matrix λ or in the neutrino mixing matrix via a measurement of γ?*

Neutrino evolution in the sun. The evolution of the neutrino state produced in the core of the sun under the influence of the solar magnetic field and matter effects is governed by the Schrödinger-like equation [7]

$$i \frac{d}{dz} \begin{pmatrix} \varphi_- \\ \varphi_+ \end{pmatrix} = H_{\rm eff} \begin{pmatrix} \varphi_- \\ \varphi_+ \end{pmatrix} \quad \text{with} \quad (11)$$

$$H_{\rm eff} \equiv \begin{pmatrix} V_L + U_L \frac{\hat{m}^2}{2\omega} U_L^\dagger & -B_+ \lambda^\dagger \\ -B_- \lambda & V_R + U_R \frac{\hat{m}^2}{2\omega} U_R^\dagger \end{pmatrix} .$$

In this equation, φ_- and φ_+ denote the vectors of neutrino flavour wave functions corresponding to negative and positive helicity, respectively. $V_L = \sqrt{2} G_F \mathrm{diag}(n_e - n_n/2, -n_n/2, -n_n/2, 0, \ldots)$

is the matter potential where n_e (n_n) is the electron (neutron) density in the sun and $V_R = 0\,(-V_L)$ for Dirac (Majorana) neutrinos. The diagonal matrix of neutrino masses is denoted by $\hat{m}$ and U_L is the unitary mixing matrix connecting left-handed flavour and mass eigenfields. U_R is an arbitrary unitary matrix for Dirac neutrinos and $U_R = U_L^*$ in the Majorana case. Finally, $B_\pm = B_x \pm iB_y$ where B_x and B_y are the components of the solar magnetic field orthogonal to the neutrino momentum.

Neutrinos are produced as electron neutrinos in the sun at the coordinate z_0 and are detected on earth at z_1. Hence we express the initial condition as $\varphi_-^T(z_0) = (1, 0, \ldots)$, $\varphi_+^T(z_0) = (0, \ldots)$ and for the neutrino state at the detector, Eq.(6), we have $a_\mp \equiv \varphi_\mp(z_1)$. For a given magnetic field along the neutrino path in the sun, the neutrino state described by the vectors $a_\mp$ can in principle be obtained by solving Eq.(11), as a function of neutrino MMs, EDMs, masses and mixing parameters. These flavour vectors $a_\mp$ have to be used in the cross section for elastic νe^- scattering of solar neutrinos.

Phase counting. Let us first consider a single Dirac neutrino. In this simplest case there is only one complex phase in the problem, which is the phase δ defined through $\mu + id = \sqrt{\mu^2 + d^2}\, e^{i\delta}$ where μ and d are real numbers in this case. Now, the evolution Eq.(11) leads to $a'_+ = e^{i\delta} a_+$ such that a_- and a'_+ as well as the angle γ in the interference cross section (10) depend only on $\sqrt{\mu^2 + d^2}$. Hence, the phase δ disappears from the problem. This means that it is not possible to distinguish between a Dirac MM and EDM via a measurement of γ. A nonzero d leads to CP violation at the level of the Lagrangian. However, the above consideration implies that no CP violating effects can be observed in elastic νe^- scattering of solar neutrinos.

In the second simple case of two Majorana neutrinos there are two phases in the problem: one in the transition moment $\mu - id$ in the matrix λ (μ and d imaginary) and one phase in the mixing matrix (Majorana phase). One can show with arguments similar to the one Dirac case that both phases can be absorbed through redefinitions. Again it is not possible to distinguish between a MM and an EDM and no CP violating effects show up.

Table 1
Number of complex phases for n flavours:

	Dirac	Majorana
mix. matrix	$(n-1)(n-2)/2$	$n(n-1)/2$
λ	n^2	$n(n-1)/2$
azim. asym.	$(3n-2)(n-1)/2$	$n(n-2)$

Now we come to the general case of n neutrino flavours. In the first two lines of Table 1 we give the numbers of complex phases in the mixing matrix and in λ at the level of the Lagrangian, in a phase convention where as much phases as possible are removed from the mixing matrix. However, not all of these phases show up in elastic νe^- scattering of solar neutrinos because of the relevant physical approximations: neutrino masses enter in the evolution equation (11) only via the terms $U_L \hat{m}^2 U_L^\dagger$, $U_R \hat{m}^2 U_R^\dagger$ and are neglected in the cross section. Therefore, in the physical situation under consideration there is more phase freedom which can be used to reduce the number of phases in the problem. Moreover, complex phases can be shifted from the mixing matrix to λ and vice versa [5]. In the last line of Table 1 we show the total number of physical phases relevant for the azimuthal asymmetry in elastic νe^- scattering of solar neutrinos.

Decoherence effects. Here we consider the effect of neutrino oscillations and averaging over the neutrino energy on the solar neutrino state arriving at the earth.

The neutrino state undergoes vacuum oscillations between the sun and the earth. Therefore, denoting the values of $\varphi_\mp$ (11) at the edge of the sun by $b_\mp$, we have $a_- = U_L \exp\left(-i\hat{m}^2 L/2\omega\right) U_L^\dagger\, b_-$ and $a_+ = U_R \exp\left(-i\hat{m}^2 L/2\omega\right) U_R^\dagger\, b_+$. Here $L \approx 1.5 \times 10^{11}$ m is the distance between the sun and the earth. Now the crucial point is that, according to the quadratic appearance of $a_\mp$ in the cross section, the phase factors $e^{\pm i\varphi_{jk}}$ are important with $\varphi_{jk} = 2\pi L/\ell_{jk}$ where $\ell_{jk} = 4\pi\omega/\Delta m^2_{jk}$ is an oscillation length with $\Delta m^2_{jk} = m_j^2 - m_k^2 > 0$. The phases vary with energy as

$$\delta\varphi_{jk} = \frac{\Delta m^2_{jk} L}{2\omega}\frac{\delta\omega}{\omega} = 2\pi\frac{L}{\ell_{jk}}\frac{\delta\omega}{\omega}\,. \qquad (12)$$

Hence, integration over energy intervals such $\delta\omega \gg \omega\,\ell_{jk}/L\ \forall j,k$ leads to an averaging of the oscillations, which can formally be expressed as $\left\langle e^{\pm i\varphi_{jk}}\right\rangle = \delta_{jk}$, where δ_{jk} is the Kronecker delta. Numerically, we have

$$\frac{\ell_{jk}}{L} \approx 1.7\times 10^{-11}\,\frac{\omega(\mathrm{MeV})}{\Delta m^2_{jk}(\mathrm{eV}^2)}\,. \qquad (13)$$

If we consider, for example, $\Delta m^2 \sim 10^{-8}$ eV^2 allowed by the RSFP scenario [2] and $\omega \approx 0.27$ MeV, the average energy of the pp neutrinos, we find $\ell/L \sim 5\times 10^{-4}$, where ℓ is the oscillation length corresponding to Δm^2. Therefore, to avoid the averaging one would have to measure the neutrino energy with an accuracy better than $\delta\omega/\omega \sim 10^{-4}$, which seems rather impossible. The energy averaging of the vacuum oscillations is equivalent to consider the neutrino state arriving at the earth as an incoherent mixture of mass eigenstates.

The expressions for the general energy averaged cross sections are given in Ref.[5]. Here we want to discuss the effect of decoherence for the 2-Majorana neutrino case. For total incoherence the interference cross section is proportional to

$$\left\langle\frac{d^2\sigma^M_{\mathrm{int}}}{dT d\phi}\right\rangle \propto f\,\frac{T}{\omega}\,\sin 2\theta\cos(\phi-\gamma) \qquad (14)$$

where f is given after Eq.(9). Again all complex phases disappear, especially γ does not depend on the phase of $\mu - id$. There are two important implications of relation (14): (i) The dependence on the electron recoil energy of this expression is very different from the corresponding term in the case of full coherence and the Dirac terms with and without coherence, because the recoil energy T drops out of the product fT. (This is also true for an arbitrary number of Majorana neutrinos.) (ii) Expression (14) is proportional to the mixing angle $\sin 2\theta$. Large values for $\sin 2\theta$ are disfavoured in the RSFP scenario [2] and by the non-observation of electron antineutrinos in Super-Kamiokande [8] and hence the asymmetry is suppressed.

These arguments suggest that a significant asymmetry measured in an experiment is unlikely to result from a 2-Majorana neutrino scenario, except for very small mass-squared differences ($\Delta m^2 < 10^{-11}$ eV^2). Of course, it could result from Dirac diagonal moments. In this case the states of negative and positive helicity belong to the same mass eigenvalue and no averaging due to oscillations is possible.

Conclusions. We have considered the possibility to investigate neutrino properties using the azimuthal asymmetry in elastic νe^- scattering of solar neutrinos. We have shown that it is not possible to distinguish MMs and EDMs for 1-Dirac and 2-Majorana neutrinos and no CP violation will show up in these cases. For n neutrino flavours there is a phase freedom because of the physically motivated approximations. This allows to eliminate complex phases in the mixing matrix and the MM/EDM matrix. Furthermore we have shown that energy averaging is important and leads to a suppression of the azimuthal asymmetry in the 2-Majorana case.

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